Math 43 Midterm 2 Review

[1] Eliminate the parameter to find rectangular equations corresponding to the following parametric equations.
For [a] and [d], write y as a function of x.

[a] $x = \dfrac{t}{1-t}$

$y = \dfrac{t-1}{1+t}$

[b] $x = 3 + 5\tan t$

$y = 4 + 2\sec t$

[c] $x = 8 + 6\cos t$

$y = 7 - \sin t$

[d] $x = 5\ln 4t$

$y = 2t^3$

[2] Find parametric equations for the following curves using templates from your lecture notes, textbook and exercises.

[a] the line through $(-3, -6)$ and $(7, -2)$

[b] the circle with $(-3, -6)$ and $(7, -2)$ as endpoints of a diameter

[c] the ellipse with $(-3, -6)$ and $(7, -6)$ as foci, and $(2, -2)$ as one endpoint of the minor axis

[d] the hyperbola with $(-3, -6)$ and $(7, -6)$ as vertices, and $(-5, -6)$ as one focus

[e] the portion of the graph of $y = 2x^4 - 3x^3 + 1$ from $(-1, 6)$ to $(2, 9)$

[3] A circular skating rink has a radius of 50 feet. AJ is standing 70 feet to the left of the center of the rink. BJ enters the rink on the opposite side from AJ, and skates counterclockwise around the rink, making one full circle every 20 seconds. If AJ is standing at the origin, find parametric equations for BJ's position at time t seconds after entering the rink.

[4] Find the value of $\displaystyle\sum_{n=3}^{8} (-1)^n n(n-4)$.

[5] Calculate $\dbinom{200}{4}$.

[6] Write the repeating decimal $0.4\overline{72}$ as a simplified fraction. **NOTE: Only the 72 is repeated.**

[7] Use sigma notation to write the series $\dfrac{1}{7\cdot 3} + \dfrac{1}{4\cdot 6} + \dfrac{1}{1\cdot 12} + \cdots - \dfrac{1}{17\cdot 768}$.

[8] Find the coefficient of x^{34} in the expansion of $(2x^5 - 3x^2)^{11}$.

[9] Find the value of $\displaystyle\sum_{n=3}^{\infty} 4(0.97)^{2n-1}$. **HINT: Write out the first few terms first.**

[10] Find the first 5 terms of the sequence defined recursively by $a_n = 2a_{n-1} - 3, \quad a_1 = 4$.
Is the sequence arithmetic, geometric or neither ? Explain how you arrived at your conclusion.

[11] Use Pascal's triangle and the Binomial Theorem to expand and simplify

[a] $(3x - 2y)^6$

[b] $\left(\sqrt{x} - \dfrac{2}{x}\right)^4$

[12] EJ bought a new car in 1998. The registration fee was $\$800$ that year. Each year, the registration fee decreased by 10%. The car was eventually sold for scrap in the year when its registration fees were $\$3.34$. What year was EJ's car sold for scrap ?

[13] Prove by mathematical induction:

[a] $1^3 + 2^3 + 3^3 + \cdots + n^3 = \dfrac{n^2(n+1)^2}{4}$

for all integers $n \geq 1$

[b] $n! > 3^{n-1}$

for all integers $n \geq 5$

[14]   Find the sum of the series $-73 - 66 - 59 - 52 \cdots + 529$.

[15]   CJ and DJ both just graduated from college and started new jobs. Neither could afford the market rate for apartment rentals, so they worked out deals with their landlords. CJ agreed to pay $400 rent the first month, and each month after, $7 more rent than the previous month. DJ agreed to pay $380 rent the first month, and each month after, 2% more rent than the previous month. After 2 years, who will have paid more rent altogether, and by how much ?